Numerical Analysis of Inter-Electrode Capacitance of Vacuum Micro-Electronics Devices

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Abstract

In this paper, calculation of inter-electrode capacitance per unit length of vacuum micro-electronics devices is presented using finite difference method (FDM). A comparison of the numerical solution with the CST/COMSOL simulated results of the same structure is also presented in this paper. The numerical results thus obtained closely agree with simulation results

1. Introduction

Vacuum tubes are used for a few niche applications such as premier sound systems and high-power radio base stations, high power high frequency microwave sources, high power X-Ray sources etc. due to high gain, fast speed, and superior distortion immunity. However, vacuum tubes have been replaced by solid-state devices such as transistors due to their poor reliability and high-power consumption. In the solid-state device's electrons have to propagate through a solid medium, they are very much prone to radiation damage. Also, high temperature operation is not possible as the medium property varies with temperature. The electrons, while travelling through the solid media, suffer collisions (crystal-lattice scattering), resulting in noisy and distorted output.

With the advent of micro fabrication technologies, the transistors could be fabricated at lower cost, smaller sizes, superior lifetime higher efficiency, ruggedness and reliability. Therefore, researchers have come up with the concept of vacuum channel transistor which combines the best of vacuum tubes and modern semiconductors into a single device. It consists of a field electron source, an anode electrode and the gate electrode. When the voltage is applied across the emitter and the anode, due to field emission the electrons are emitted. The gate controls the current flow through the vacuum channel. The proposed device equivalent circuit diagram superimposed on the basic structure of a vacuum micro electronic device and the physical dimension (in mm) are shown in figure 1 [1]. In the notations, R is resistance and C is inter-electrode capacitance of the device.

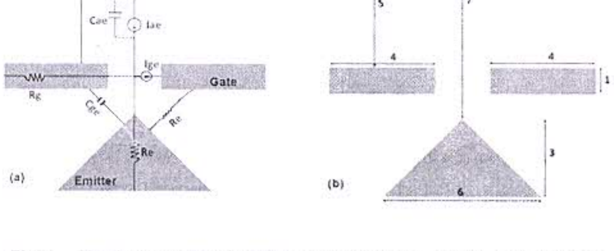

Fig 1. Physical model of VMD. (a) Equivalent circuit, (b) Physical dimension (in mm)

The emission current due to the electric field created by the potential difference between the Emitter tip and the gate, can be characterized by Fowler-Nordheim equation as

$$I = aV^2 exp\left(-\frac{b}{V}\right) \qquad (1).$$

Where, a = 1.54×10^{-6} (α / ϕ) $\beta^2 \exp(9.89/\sqrt{\phi})$ and b = 6.53×10^{-7} ($\varphi^{3/2}/\beta$), α is the emitting area in (cm^2), φ is the work function of tip material in (ev), and β is the geometric factor in (cm^{-1}) depending on the electrode configuration.

The cut-off frequency f_T can be defined as a frequency at which the common source short circuit current gain comes to unity and it can be written as:

$$f_T = \frac{g_m}{2\pi C_{GE}} \qquad (2).$$

where, g_m is the small signal transconductance and C_{GE} is inter electrode capacitance between emitter and gate as shown in figure 1. g_m can be defined as

$$g_m = \frac{\delta I}{\delta V} = \left(\frac{I}{V}\right)\left(2 + \frac{b}{V}\right) \qquad (3)$$

In order to enhance cut off frequency one can either increase g_m or decrease C_{GE}. In a field emitter array, g_m and C_{GE} are proportional to the number of tips present in that array. Therefore, simple increasing of the number of tips does not significantly improve the cut-off frequency of the device. For improving f_T, g_m should be increased by optimizing the tip emission characteristics or by choosing the operating point of higher gate bias voltage or the capacitance value should be decreased by the increase of the emitter packing density [1]

In order to optimize all the parameters, a complete analytical modelling is required. In this paper an analytical model for estimation of inter-electrode capacitance, which is one of the key parameters, will be estimated numerically. Several methods were used by various researchers. Capacitance of rectangular parallel plate capacitors was obtained by the method of subareas by Reitan [2]. Harrington et. al. [3] used projection method, Mautz [4] used point-matching method and Benedek [5] used projective method for estimation of capacitance. In this paper Finite Difference Method (FDM) was used to determine capacitance per unit length of vacuum micro-electronics devices. The obtained result was verified using simulation results.

2. Numerical Computation using FDM

Inter electrode capacitance can be estimated using following equation

$$C = \frac{Q}{V} = \frac{\oint \rho_s}{V} \qquad (4).$$

Where Q is the total charge accumulated on the surface of the electrode, ρ_s is the surface charge density, and V is the potential difference between two electrodes.
ρ_s can be written as:

$$\rho_s = \nabla \cdot \boldsymbol{D} = \varepsilon \nabla \cdot \boldsymbol{E} = \boldsymbol{\varepsilon}_r \varepsilon_0 \nabla \cdot \boldsymbol{E} \qquad (5).$$

Where, D is the displacement current, E is the electric field, ε_0 is the permittivity of the free space, and ε_r is the dielectric permittivity of the medium between the electrode.
In order to calculate the electric field and charge density distribution, Poisson's equation has solved using FDM technique. Dirichlet boundary conditions is used for calculation of potential distribution. Solutions obtained by FDM are approximate only and the error in the solution can be minimized by very fine discretization of the domain.
The basic idea of FDM is to discretize solution domain into finite discrete points. To make the analysis simple, the entire solution domain is divided into square grid shown in Fig. 2, The partial differential equations is then represented with a set of difference equations.

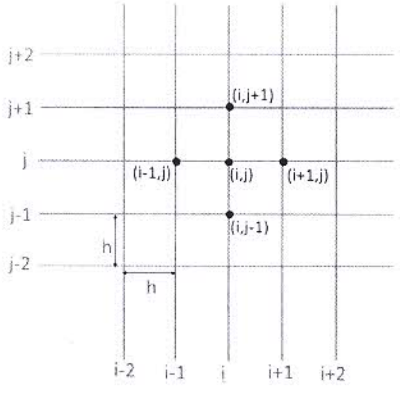

Fig. 2. Grid nodes in X-Y plane subdivided by square meshes in the Cartesian coordinates

Fig. 2 depicts the mesh structure of the analysis region. Blue lines indicate the and the distance between the grid lines are called mesh lengths. Intersection of grid lines showing in black point in figure 2 is called a node. The continuous function is replaced by a great number of discretized values at these nodes after the domain is discretized into grids.
Poisons equation in 2D Cartesian coordinate can be written as:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = -\frac{\rho}{\varepsilon} \qquad (6).$$

The solution of above equation can be written in differential form as:

$$V_{i,j} = \frac{1}{4}\left(V_{i,j+1} + V_{i,j-1} + V_{i+1,j} + V_{i-1,j} + \frac{h^2\rho}{\varepsilon}\right) \qquad (7).$$

The domain was divided into 160 x 160 matrix and the solution of the Eq's. (7) was obtained using MATLAB after several iterations. Initial voltage at the emitter, gate, and anode were defined as 0 V, 10 V, and 50 V respectively. After obtaining potential distribution in the entire domain the inter-electrode capacitance was calculated using equation (6), (5), and (4).

3. Results and Discussion

In order to verify numerical result, the entire geometry was simulated using Comsol Multiphysics and results were compared. In this analysis the detail dimension and potential of electrode is shown in figure 1. The equipotential lines and electric field distribution both calculated and simulation is shown in Figure 3. It can be seen that the potential and electric fields are bended at the edges of the electrodes. The value of the electric field is also greater in the sharp edges of the electrodes. A comparison data derived using code as well as simulation is shown in table I. They are found to be in close agreement.

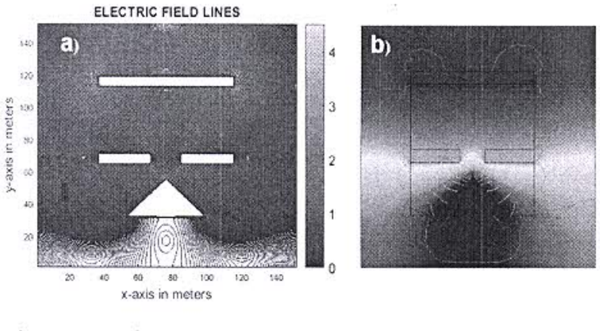

Fig. 3. Equipotential lines and field distribution obtained using (a) FDM technique, (b) Comsol Multiphysics.

Table I: Comparative study of capacitance per unit cell

Inter Electrode Capacitance	**Capacitance per unit length (F/m)**	
	Numerical	*Simulation*
C_{GE}	1.0321 e-10	0.4507e-10

4. Conclusion

In this work inter-electrode capacitance per unit length of vacuum micro-electronics devices was numerically calculated successfully using FDM technique. The same model was simulated using CST in order to verify the numerical results. It shows that, numerical results closely match with simulation data. After introducing emission model in this code, transconductance (g_m) of the transistor can be estimated numerically using equation (3). By knowing the value of transconductance (g_m) and inter-electrode capacitance cut-off frequency of a transistor can be estimated using equation (2). Hence, it is proposed to use this code for complete design of a vacuum micro-electronic devices.

5. References

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