A Robust H-infinity Based Depth Control of an Autonomous Underwater Vehicle

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***Abstract*—This paper presents a robust H-infinity based control methodology for an Autonomous Underwater Vehicle(AUV). The kinematics and dynamics of an AUV is described using six degree of freedom differential equations of motion using body and earth-fixed frame of references. Due to hydrodynamic forces, these equations are highly coupled and non-linear. From the practical point of view it is essential to consider a reduced order model for efficient controller design. Hence the system is commonly subdivided into smaller subsystems, like depth, steering (or yawing) and speed subsystems, which are considered to be mutually non-interactive from the controller design perspective. In this study a reduced order model was derived using the depth plane dynamics of the vehicle. The working environment of an AUV is vastly uncertain due to varying environmental conditions, thereby demanding a robust controller which has the ability to adapt to these uncertainties and provide stabilizing effect irrespective of the change in the surrounding conditions. The proposed H-infinity controller takes into account the uncertainties in the hydrodynamic parameters which arise due to changing operating conditions and provides suitable control action for desired set point tracking as well as disturbance rejection. The altitude of the vehicle is strongly dependent on the pitch angle, and the controller presented here takes care of both the pitch and depth plane dynamics. The mixed sensitivity approach for H-infinity controller design is followed, and the efficacy of the controller is compared with Linear Quadratic Gaussian(LQG) controller and the Mixed H2/H-infinity controller. The controller design and simulation has been done in Matlab, and the simulated results provide satisfactory results, for disturbance rejection and set point tracking for the H-infinity controller in presence of hydrodynamic parametric uncertainties.**

Keywords*—Autonomous Underwater Vehicle (AUV), H-infinity control, Linear Quadratic Gaussian(LQG) control, mixed H2/H-infinity control, six-degrees of freedom.

1. INTRODUCTION

Autonomous Underwater Vehicles are being increasingly used in ocean survey, which includes ocean floor mapping, survey of underwater biotic environment and has become especially useful for the oil and gas industry. For an underwater vehicle to be truly autonomous, it is essential that it has good adaptability to changing operating conditions, is not affected by external disturbances and it can efficiently complete its pre-programmed tasks, without any kind of manual intervention. To meet the afore mentioned requirements, one of the most important task is the design of a robust and efficient controller which can compensate for the hydrodynamic parametric uncertainties which affects the vehicle dynamics and keeps the system stable under all operating conditions, as well as provide good set point and trajectory tracking capability.

Many types of controllers and control strategies have been applied and proposed in the past for controlling the dynamics of underwater vehicles authors. To compensate the inherent non-linearities in the dynamic equations of the vehicle, sliding mode control[4], and back stepping control methodology based on Lyapunov stability theory have been applied[8]. Intelligent control system based on Fuzzy Logic has been proposed in [5] as this technique does not depend much on the system model. The inherent capability of neural networks as function approximators and classifiers and their capability to deal with uncertainties have also made them a candidate for use as control strategy in underwater vehicles [7].

The H-infinity controller aims at minimizing the infinity norm of the system, that is, it reduces the maximum disturbance affecting the system. H-infinity controllerdesign for autonomous underwater vehicle has been discussed in [10],[11]. In case of underwater vehicles, sensor noise also forms a source of disturbance and uncertainty which affects the vehicle. Linear Quadratic Gaussian (LQG) control exhibits good noise rejection capability as well as robustness and has been proposed in[9]. In this paper a mixed H2-H-infinity control strategy has been proposed, which provides both noise and disturbance(arising due to parametric uncertainties) rejection along with good set point tracking.

The paper has been arranged in the following subsections. Section 2 presents mathematical modeling of the system, taking into account both the kinematics and dynamics part. Model Uncertainties is explained in section 3. H-infinity control theory has been discussed in section 4. The mixed sensitivity approach to H-infinity controller design and loop shaping is presented in Section 5. In Section 6 the simulations results have been the provided and the concluding section involves discussion of the results obtained and the conclusion derived at, from the studies conducted for this paper.

2. MATHEMATICAL MODELING OF AUV

The notation used in this paper is in accordance to SNAME 1950[1]. The six degree of freedom is based on the Newton-Euler equations as described in [2]. Generally two coordinate frames are considered while modeling an AUV. One is the inertial or the earth-fixed which is considered to be stationary and the other is the body-fixed frame, which is assumed to be fixed to the body of the vehicle and moves along with it. The earth-fixed and the body-fixed frame is represented by two vectors η andv respectively, consisting of six components each. The translational and angular coordinates of the vehicle along the three principal coordinate axes X, Y and Z are expressed in terms of the earth fixed frame, while the linear and angular velocities of the vehicle along the three axes is expressed in terms of body-fixed coordinate frame.

$\boldsymbol{\eta}=[xyz\phi\theta\psi]^T$ is the earth fixed vector, where$\boldsymbol{\eta}_1=[xyz]^T$ are the position coordinates and $\boldsymbol{\eta}_2=[\phi\theta\psi]^T$ are the rotational coordinates.The body-fixed vector is given by $\nu=[uvwpqr]^T$, where $\nu_1=[uvw]^T$ are the translational velocities and are known as surge, sway and heave velocities and $\nu_2=[pqr]^T$ are the rotational velocities and are known as roll,pitch and yaw motions. The two coordinate frames are related by the Jacobian matrix $J(\boldsymbol{\eta}_2)$ and is given by

$$\dot{\eta} = J(\boldsymbol{\eta}_2)\nu \qquad (1)$$

Equation (1) represents the Kinematics of the underwater vehicle. The dynamics of the vehicle consists of two parts, the translational dynamics and the rotational dynamics. The translational dynamics is given by Newton's second law $F = ma$ (2)

$m$is the mass of the body and a is the acceleration of the body.

The rotational dynamics is given by Euler's equation, which states that rate of change of angular momentum are equal to the moment applied.

$$M_c = I_c\dot{\omega} + \omega \times I_c \qquad (3)$$

In equations (2) and (3) $F$and $M_c$are the external forces and moments which acts on the body. These include gravitational, buoyancy, propulsive, damping and control forces. Based on the above principal force and moment equations, the general six degrees of freedom equations of motion of an autonomous underwater vehicle is given by

$$m[\dot{u} - vr + wq - x_g(q^2 + r^2) + y_g(pq - \dot{r}) + z_g(pr + \dot{q}) = \sum X_{ext}$$
$$m[\dot{v} - wp + ur - y_g(r^2 + p^2) + z_g(qr - \dot{p}) + x_g(qp + \dot{r}) = \sum Y_{ext}$$
$$m[\dot{w} - uq + vp - z_g(p^2 + q^2) + x_g(rp - \dot{q}) + y_g(rq + \dot{p}) = \sum Z_{ext}$$
$$I_{xx}\dot{p} + (I_{zz} - I_{yy})qr + m[y_g(\dot{w} - uq + vp) - z_g(\dot{v} - wp + ur)] = \sum K_{ext}$$
$$I_{yy}\dot{q} + (I_{xx} - I_{zz})rp + m[z_g(\dot{u} - vr + wq) - x_g(\dot{w} - uq + vp)] = \sum M_{ext}$$
$$I_{zz}\dot{r} + (I_{yy} - I_{xx})pq + m[x_g(\dot{v} - wp + ur) - y_g(\dot{u} - vr + wq)] = \sum N_{ext}$$

(4)

Reduced Order Model

In this paper the depth plane motion of the AUV is considered. The depth of the vehicle depends on the pitch angle, and required depth of the vehicle can be achieved by effective controlling of the pitch angle. Considering the depth plane dynamics only, and neglecting all out of plane terms, the reduced order state space model of the system can be written as

$$\begin{bmatrix} I_{yy} - M_{\dot{q}} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{q} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} -M_q & 0 & -M_\theta \\ 0 & 0 & U \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ z \\ \theta \end{bmatrix} = \begin{bmatrix} M_{\delta s} \\ 0 \\ 0 \end{bmatrix} \qquad (5)$$

In equation (4) I_{yy} is the moment of inertia about Y axis, M_θ,M_q, $M_{\dot{q}}$ are hydrodynamic parameters defined in TABLE 1 this can be re-arranged as

$$\begin{bmatrix} \dot{q} \\ \dot{z} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{M_q}{I_{yy} - M_{\dot{q}}} & 0 & \frac{M_\theta}{I_{yy} - M_{\dot{q}}} \\ 0 & 0 & -U \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} q \\ z \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{M_{\delta s}}{I_{yy} - M_{\dot{q}}} \\ 0 \\ 0 \end{bmatrix} \delta s \qquad (6)$$

$\delta s$is the input given to the system which may be step, sinusoidal or any other input.
This is of the form x□(t)=Ax(t)+Bu(t)
Where

$$A = \begin{bmatrix} -0.82 & 0.00 & -0.69 \\ 0.00 & 0.00 & -1.54 \\ 1.00 & 0.00 & 0.00 \end{bmatrix}$$

$$B = \begin{bmatrix} -4.16 \\ 0.00 \\ 0.00 \end{bmatrix} \qquad (7)$$

U is the linear velocity of the vehicle (1m/s).
The transfer function for the pitch control is obtained from the above state space equation as

$$\frac{\theta(s)}{\delta s(s)} = \frac{\frac{M_{\delta s}}{I_{yy} - M_{\dot{q}}}}{s^2 - \frac{M_q}{I_{yy} - M_{\dot{q}}}s - \frac{M_\theta}{I_{yy} - M_{\dot{q}}}} \qquad (8)$$

From the table (1) we derive the transfer function as

$$\frac{\theta(s)}{\delta s(s)} = \frac{-4.16}{s^2 + 0.82s + 0.69} \qquad (9)$$

And the depth loop equation is

$$\frac{z(s)}{\theta(s)} = -\frac{U}{s} \qquad (10)$$

where$U$is the linear velocity of the vehicle. Usually the velocity of underwater vehicles vary between 1m/s to 4m/s. In this study the velocity of the vehicle is considered to be 1m/s. So the depth plane equation can be written as

$$\frac{z(s)}{\theta(s)} = -\frac{1}{s} \qquad (11)$$

TABLE 1.HYDRODYNAMIC PARAMETER FROM [3].

Parameter	Value	Units	Description

Iyy	3.45	kgm²	MI along y axis about COM
$M_{\dot{q}}$	-4.88	kgm²	Added Mass term
M_q	-6.87	kgm²/s	Combined term
M_θ	-5.77	kgm²/s²	Hydrostatic moment (pitch axis)
$M_{\delta s}$	-3.46	kgm²/s²	Fin Lift moment

3. Model Uncertainty

The concept of uncertain dynamical systems and the various ways to compensate for it is central to the idea of robust control theory. For practical controller design purposes complex dynamical systems are approximated by simpler and linearized models. The gap between the working models and the actual dynamical systems is known as the model uncertainty. Another cause of uncertainty is the imperfect knowledge of some of the model parameters, or the alteration of their behavior with change in operating conditions. The dynamical model of underwater vehicles are highly coupled and non-linear, and for all practical purposes a linearized and reduced order model is considered. As a result some of the high frequency components are left unmodeled. This may act as a source of disturbance during actual operation. Another source of disturbance affecting underwater vehicles is parametric uncertainties. Hydrodynamic parameters used in the system model are extremely difficult to determine with a high degree of accuracy, and may change with change in operating conditions, like density and salinity of water. H-infinity based controller is best suited to handle these uncertainties. Uncertainties can be expressed either in polytopic form or in linear fractional model form. In the polytopicform the uncertain parameters vary within a given polytope of values whereas in the linear fractional model, the uncertainty part is drawn out of the nominal plant and separately represented. Figure1 shows a standard linear fractional model used in robust control analysis. G represents the nominal plant, K the controller and Δ represents the uncertainty structure affecting the plant. Generally Δ consists of diagonal matrices, each representing a particular parametric uncertainty. $\Delta = \mathrm{diag}\ (\Delta_1, \Delta_2, \ldots\ldots\ldots, \Delta_n)$. w represents the input disturbance signal and z is the corresponding disturbance output due to uncertainty Δ. The aim of the controller is to provide a control input u, such that it minimizes the gain from input w to output z in presence of system uncertainty Δ.

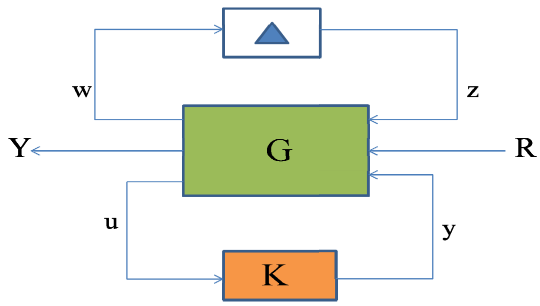

Figure 1:Linear fractional model of an uncertain system

4. H-infinity Control

The H-infinity full information control problem is to find a feedback controller, using the state and disturbance, that minimizes the closed loop infinity norm and is given by

$$\|G_{zw}\|_\infty = sup_{\|w(t)\|_2} \frac{\|z(t)\|_2}{\|w(t)\|_2} \qquad (12)$$

where$z(t)$ is the output signal generated by the system and $w(t)$ is the input disturbance signal to the system. The aim of the controller is to minimize the maximum gain of the system in the energy or L_2norm sense. Relating to the above equation$w(t)$ is considered to be an arbitrary signal in L_2 space and the task of the controller is to minimize its worst case effect on the energy of$z(t)$. The 2-norm of a signal is given as

$$\|z(t)\|_2 = \left(\int_{-\infty}^{\infty} e^T\ (t) e(t) dt \right)^{\frac{1}{2}} \qquad (13)$$

A signal is said to belong to the L_2space if the integral in equation(13) converges to a finite value.Tractable solution for equation(12) can be obtained by considering a bound on the closed loop infinity norm, and can be written as

$$\|G_{zw}\|_\infty = sup_{\|w(t)\|_2} \frac{\|z(t)\|_2}{\|w(t)\|_2} < \gamma \qquad (14)$$

5. Mixed Sensitivity Approach

The H-infinity problem is basically an optimization problem, in which we have to optimize between the input control energy and the output disturbance energy. The objective is to minimize the disturbance output, using minimum control energy. However these two signals are opposing in nature, and minimizing one leads to maximization of the other signal. Lower the control input greater is the input to output disturbance gain, and increase in the control input leads to a decrease in the disturbance output. The mixed sensitivity approach uses the penalty function method to solve the H-infinity control problem, in which both the disturbance and the control signal are attached with a penalizing function and the aim is to reduce the infinity norm of the linear combination of the two functions.

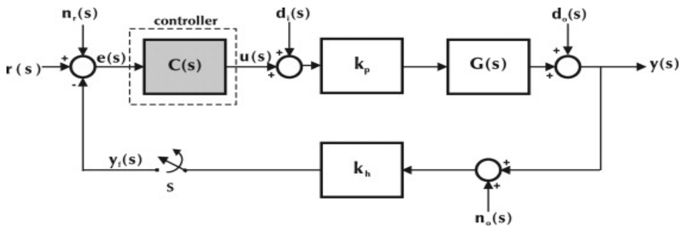

Figure 2: Closed loop control system with disturbances

Figure2 shows a closed loop control system with controller transfer function C(s), system transfer function G(s), input r(s) and output y(s). $K_p$and $K_b$are the forward and feedback loop gains. $n_i(s)$, $d_i(s)$, $d_o(s)$ are the input, control and output disturbances respectively.$n_o(s)$ is the noise affecting the output measurements. We can define three functions, Sensitivity function S(s), Transverse Sensitivity T(s) function and the Control Sensitivity function SC(s) as given below

$$S(s) = \frac{1}{1+G(s)C(s)}, \quad T(s) = 1 - S(s) = \frac{G(s)C(s)}{1+G(s)C(s)}$$

$$R(s) = SC(s) = \frac{C(s)}{1+G(s)C(s)} \quad \text{................(15)}$$

$S(s)$gives the transfer function between the error e(s) and the input r(s), $S(s) = \frac{e(s)}{r(s)}$

$T(s)$ gives the transfer function between the output y(s) and the input r(s), $T(s) = \frac{y(s)}{r(s)}$

$R(s)$gives the transfer function between the control input u(s) and the input r(s), $R(s) = \frac{u(s)}{r(s)}$

These sensitivity functions can be combined into a single infinity norm specification and can be written as

$$\|T_{y1u1}\|_\infty < 1 \qquad (16)$$

$$\text{where} T_{y1u1} \stackrel{\text{def}}{=} \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix} \qquad (17)$$

Small magnitudes of the sensitivity function $S(s)$ at low frequencies ensures good trajectory tracking characteristics. A large attenuation of the sensitivity function at low frequency also ensures good disturbance rejection characteristics. In order to achieve these characteristics W_1 is modeled as a low pass filter. At higher frequencies the important requirements are noise rejection and robust stability in the face of unmodeleddynamics. The transverse sensitivity function $T$is therefore designed to have a high gain in low frequency range. Hence W_3 is designed as a high pass filter.Over all frequencies the control sensitivity function has to be minimized. HenceW_2 is normally modeled as a low pass filter. In other words it can be said that W_1 weighs the error signal, W_2 weighs the control signal and W_3 weighs the output signal.

In this study, the depth plane dynamics of the underwater vehicle considered is of third order. The pitch control itselfis a second order system.The weighting functionsW_1 and W_3 are designed as second order low pass and high pass filters respectively. The control signal weighting function is assumed to be a small constant, to make the augmented plant full order.$W_1 = \frac{40}{s^2+4s+3}$,$W_2 = 0.001, W_3 = \frac{s^2+5s+8}{200}$ (18)

The weighting functions are chosen so that there are no imaginary axis poles or zeroes, which may make the system internally unstable.

6. Results and Discussion

In this study an uncertain model of the depth plane dynamics of the underwater vehicle was created by introducing parametric uncertainties, since in reality hydrodynamic parameters like added mass, hydrostatic moment, fin lift moment cannot be ascertained with a high degree of accuracy. In the study undertaken, it is assumed that the estimation uncertainty of the fin lift moment $M_{\delta s}$ is about twenty percent of its nominal value. The uncertainties in the values of added mass term $M_{\dot{q}}$ and hydrostatic moment M_θ is assumed to be 25 percent of their respective nominal values. The nominal values of all the parameters affecting the depth plane motion of the underwater vehicle is listed in TABLE 1. A H-infinity controller based on the mixed sensitivity approach was generated using the nominal model. The controller generated was found to be of fourth order, higher than that of the system. The closed loop system met the requirements of disturbance rejection and internal stability. The requirement that the infinity norm be less than one, was also met, verifying the fact that the controller would internally stabilize the system. With the augmented filters as given in equation(18), the optimal value of γ was found to be 0.8851. Step response of the system with twenty random parameter, whose value is varied within the specified uncertainty range is shown in Figure 3. The response of the nominal closed loop system and the uncertain system, along with the generated H-infinity controller is shown in Figure 4 and Figure5 respectively. It is seen that the system is stable, and gives very good set point tracking or regulatory control characteristics. The magnitude plot of the sensitivity function of the uncertain samples and the worst case sensitivity is shown in Figures 6 and 7 respectively. It is observed that in both the cases a peak occurs, near 10^2 rad/sec, indicating a singularity at that frequency. This is confirmed in Figure 8 which shows a hump in the singular value plot of the sensitivity function at the same frequency. So there is a chance that the system may show instability at that frequency. However the singularity can be overcome by proper choice of the weighing functionW_1, which weighs the sensitivity function.

A comparative study between H-infinity and LQG (H2), along with mixed H2-Hinfinity controller has been presented in this paper. The two norm of the system along with the controller is found to be 2.7298. The step response of the nominal system with LQG controller is shown in Figure 9. From the response it can be seen that there is an increase in overshoot when we try to reduce the two-norm of the system, instead of the infinity norm, that is when we use a H2 controller. A slight oscillatory behavior before settling is also observed. The step response for the system with mixed H2-Hinfinity controller is shown in Figure 10. No oscillatory behavior is observed in this case. However the system settles at a slightly higher value (1.5m), than the desired value. The optimal two norm and the infinity norm for the closed loop system was found out to be 1.5356 and 1.8244 respectively.

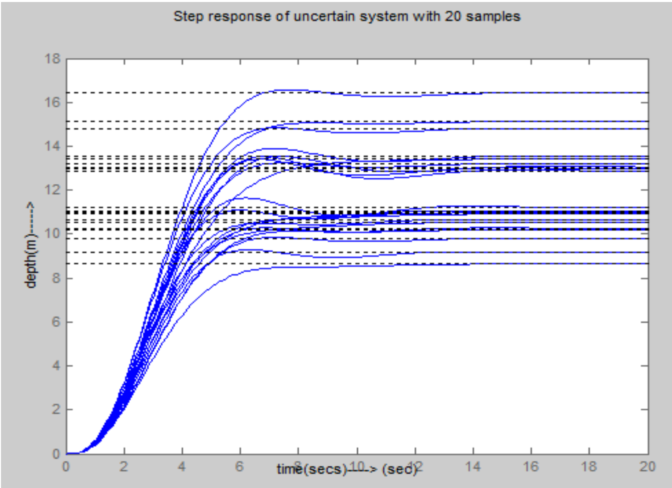

Figure 3: Step response of uncertain depth system

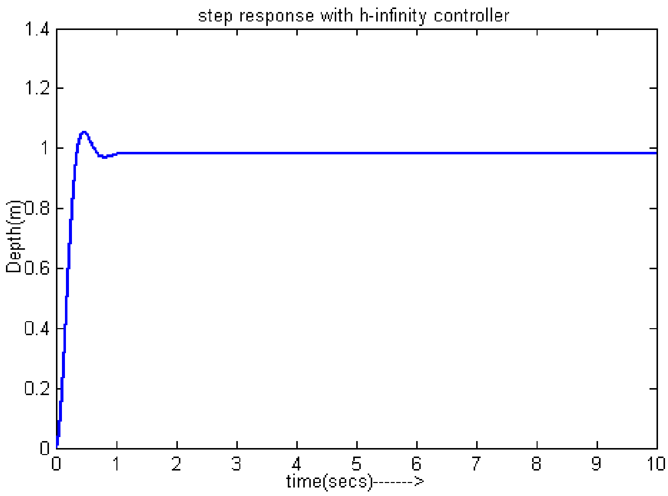

Figure 4:Step response of the nominal plant with H-infinity controller

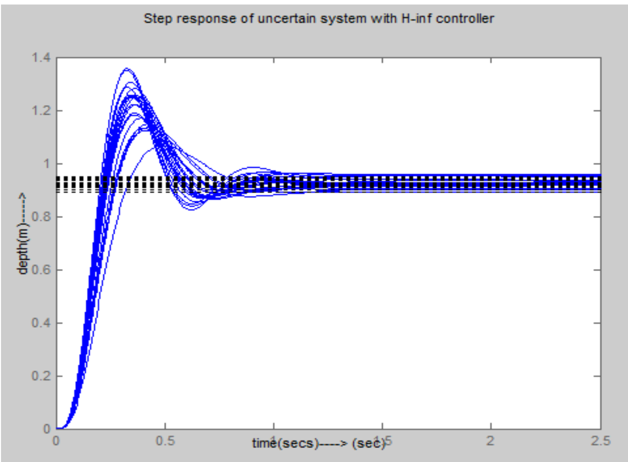

Figure 5:Step response of the uncertain depth system with H-inf controller

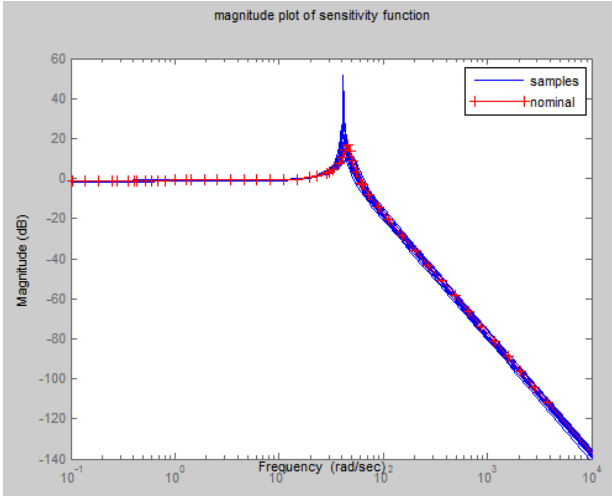

Figure 6: Magnitude plot of Sensitivity function in log scale

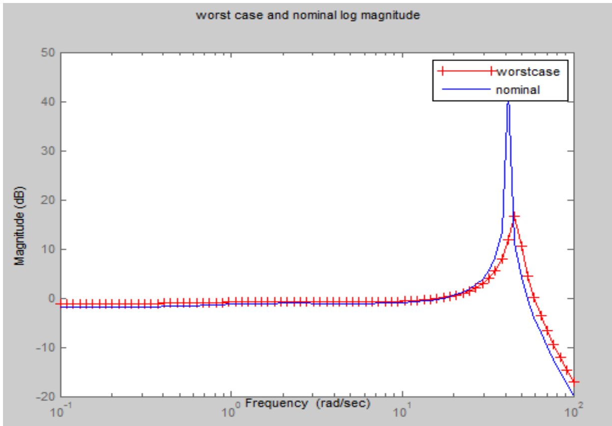

Figure 7: Magnitude of Sensitivity function in worst case

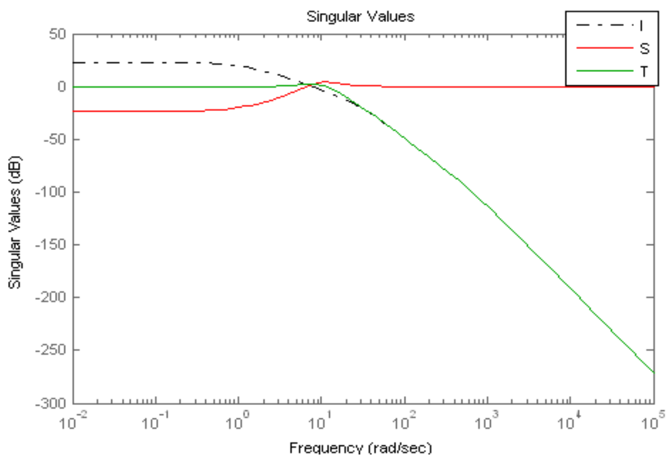

Figure 8: Singular value plot for closed loop(l) Sensitivity(S) and Transverse Sensitivity function(T)

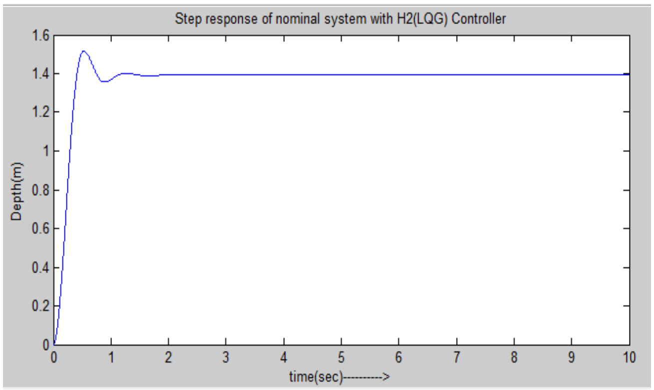

Figure 9: Step response of the nominal plant with LQG (H2) controller

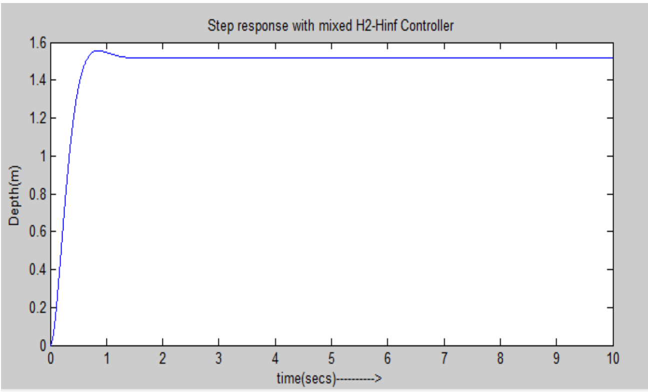

Figure 10: Step response of the nominal plant with mixed H2/H-infinity controller

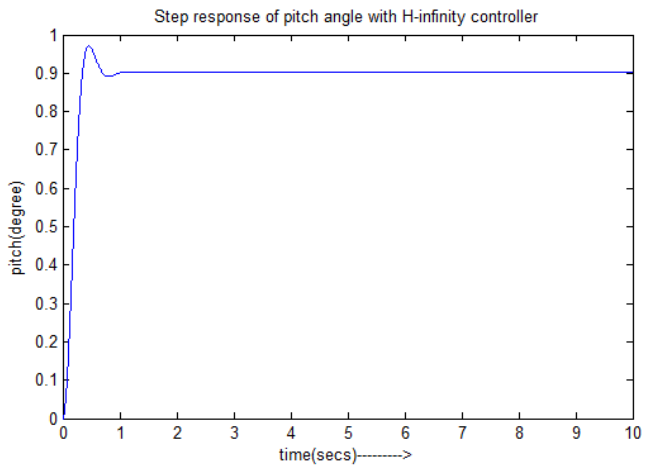

Figure 11: Pitch angle response to unit step input with H-inf controller

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