



# Design and Characterization of Rippled Wall Slow Wave Structure for X-Band PASOTRON

Minhaz Ahmad<sup>1,2</sup>, Niraj Kumar<sup>2</sup>, U N Pal<sup>1,2</sup>, Vishnu Srivastava<sup>2</sup>, Ram Prakash<sup>1,2</sup>

<sup>1</sup>Academy of Scientific and Innovative Research, New Delhi, India

<sup>2</sup>CSIR-CEERI, Pilani, Rajasthan, India.

Email: [gotominhaz@gmail.com](mailto:gotominhaz@gmail.com)

Paper # 2045  
PPPS 2013  
San Fransisco ,USA



## Motivation

An X-Band PASOTRON is being developed and a rippled wall slow wave structure is chosen for it. The design and characterization of the structure is very essential for validating the operation of device in X-Band as well as for choosing operating point for the device.

## Objective

The objective of this work is to design, develop and characterize the rippled wall slow wave structure by means of experimental and simulation studies to validate its operation in X-Band.

## Introduction

The PASOTRON mainly consists of plasma cathode electron gun (PCE-gun), periodic slow wave structure and radiating antenna [1]. An X-Band PASOTRON is being developed, in which, a sinusoidal rippled wall waveguide is chosen as slow wave structure (SWS). The reasons for this choice are that it has relatively simple structure, provides fairly effective conversion of electron-beam energy into radiation, and can be easily filled with plasma.

We present an experimental and simulation study of a slow wave structure which consists of a cylindrical waveguide with sinusoidal varying radius. A similar version of this structure was used in numerous X-band relativistic high-power backward wave oscillator experiments [2], however, until now there has not been a systematic analysis of its electromagnetic properties [3]. The purpose of this study is to experimentally determine the dispersion relation of the structure and compare it with the simulation results. The resonant method [4] we use is simple and requires less time and effort as compared to non-resonant methods [5] where we have to move a bead through the structure which makes process complicated. In the technique we use, an advantage of our structure being a resonant structure has been taken.

## Dispersion Relation

A slow wave structure having  $N$  periods when shorted at both ends and acting as a resonant structure will exhibit  $(N+1)$  resonant frequencies for  $(N+1)$  transverse magnetic axial modes. By finding the frequency and wave number at these points we can plot entire dispersion relation.

The different modes for a structure of  $N$  periods can be easily found for resonant condition.

$$N \times p = \frac{n \cdot 2\pi}{\beta}$$

$n$  = no of half wavelengths along the axis of the structure

$p$  = period of the structure

$\beta$  = axial wave number

We have chosen 4 periods of the structure in order to get 5 discrete points on the dispersion plot.

The dispersion relation helps us to find the relation between beam energy and the frequency of interaction. Thus it helps us to determine the operating point for the device.

## Experimental Arrangements

The components required are the structure with shorted ends, a probe to excite the cavity, a VNA to measure the resonant frequencies. The schematic for the experimental setup is shown in the figure 2. The dimensions for the fabricated slow wave structure are given in the table 1 and the fabricated structure in figure 1.

Mean Radius, $r_0$ (cm)	1.2
Ripple Amplitude, $r_1$ (cm)	0.5
Period (cm)	1.0

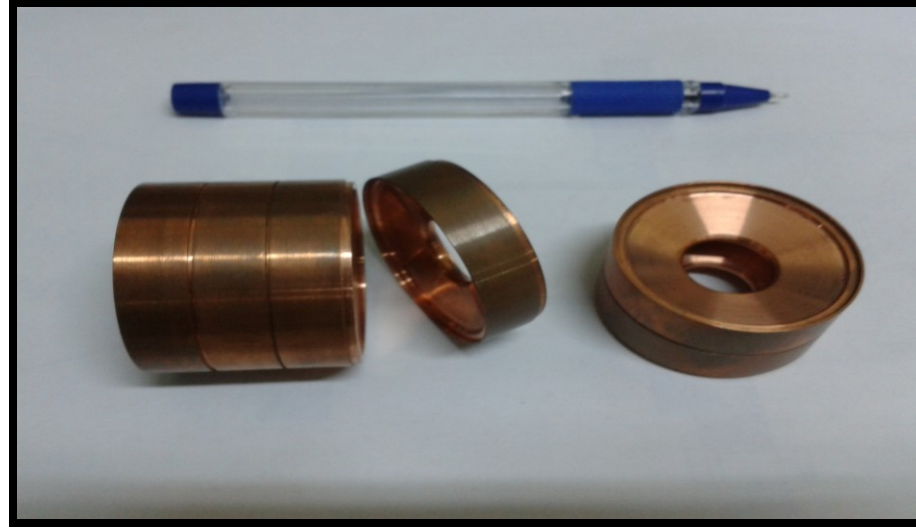


Table 1

Figure 1

Figure 2

## Linear Theory for Dispersion Relation of SWS

The axially rippled wall slow wave structure can be mathematically written as

$$r_w(z) = r_0 + r_1 \sinh_0 z = r_0(1 + \epsilon \sinh_0 z)$$

The TM modes will provide the axial perturbation in the electron beam so we are interested in the dispersion for TM modes only.

The periodicity of the waveguide allows us to expand each quantity related to TM waves, field as well as beam perturbations in a series according to Floquet's theorem.

$$E_{iz} = \sum_{n=-\infty}^{\infty} E_{zn}(r) \exp[i(k_n z - \omega t)] \quad k_n = k_0 + n h_0 \quad -h_0/2 \leq k_n \leq h_0/2$$

The field equation is then solved in cylindrical coordinates for symmetrical TM modes

$$E_{zn} = A_n J_0(\Gamma_n r) \quad 0 \leq r \leq r_b$$
$$= B_n J_0(\Gamma_n r) + C_n N_0(\Gamma_n r) \quad r_b \leq r \leq r_w$$

At beam boundary  $r_b$ ,  $E_{zn}$  must be continuous, while its derivative with  $r$  is discontinuous to a degree depending upon the amount of charge perturbation in the beam.

$$\left. \frac{dE_{zn}}{dr} \right|_{r_b^-}^{r_b^+} = i \frac{e \Gamma_n^2}{2\pi \epsilon_0 r_b k_n} \int_{r_b^-}^{r_b^+} n_b n 2\pi dr$$

Also the field must obey the condition that the component of the electric field tangential to the structure wall must vanish.

$$\left( i \frac{k_n}{\Gamma_n^2} \frac{dE_{iz}}{dr} \frac{dr_w}{dz} + E_{iz} \right)_{r=r_w} = 0$$

Applying all these conditions we represent  $B_n$  and  $C_n$  in terms of  $A_n$  and then expanding each term in the sum in fourier series we get dispersion relation as a homogeneous matrix equation.

$$D.A = 0 \quad \text{and} \quad |D| = 0 \quad \text{gives the required dispersion relation}$$

Where,

$$D_{mn} = \left( \frac{\omega^2 - k_n^2 c^2}{\Gamma_n^2 c^2} \right) \left[ I_{mn}^J - \alpha \left( \frac{\Gamma_n c}{\omega - k_n v_b} \right) \times J_0(\Gamma_n r_b) \left[ I_{mn}^J N_0(\Gamma_n r_b) - I_{mn}^N J_0(\Gamma_n r_b) \right] \right]$$

## Simulations and Measurements

The structure was simulated using CST studio simulation software and the eigen mode analysis was done to get the field pattern for different transverse magnetic ( $TM_{01}$ ) axial modes. The field pattern for the  $TM_{01}$   $\pi/2$  mode is shown in the figure 3.

Four periods of the structure was taken and shorted at both the ends by putting circular metallic plates. A hole of diameter 1 mm was drilled at the center of one circular plate so as to insert an antenna on the axis of the structure and excite the structure. Several other holes were drilled on the radial surface of the structure and the circular plates for perturbing the electric field in order to distinguish between original modes and spurious peaks and to identify electric field pattern inside the structure.

The setup was put to test by using a VNA and measuring the  $S_{11}$  parameter or the resonant peaks. The frequency range selected was from 5 GHz to 15 GHz. The resonant peaks found are shown in the figure 2. The modes were identified by introducing the perturber through different holes taking help from the simulation results.

## Results

The slow wave structure is designed in CST with parameters  $r_0=1.2$  cm,  $r_1=0.5$  cm and period =1 cm. The electric field pattern for pi mode is shown in figure 3.

The dispersion obtained from simulation and cold test are plotted together as shown in figure 4. A close agreement in results is found.

Another structure with  $r_0=1.3$  cm,  $r_1=0.1$  cm and period=1.1 cm is simulated in CST and also the dispersion is found by the linear theory. The results are plotted together as shown in figure 5 and are found to be in close agreement.

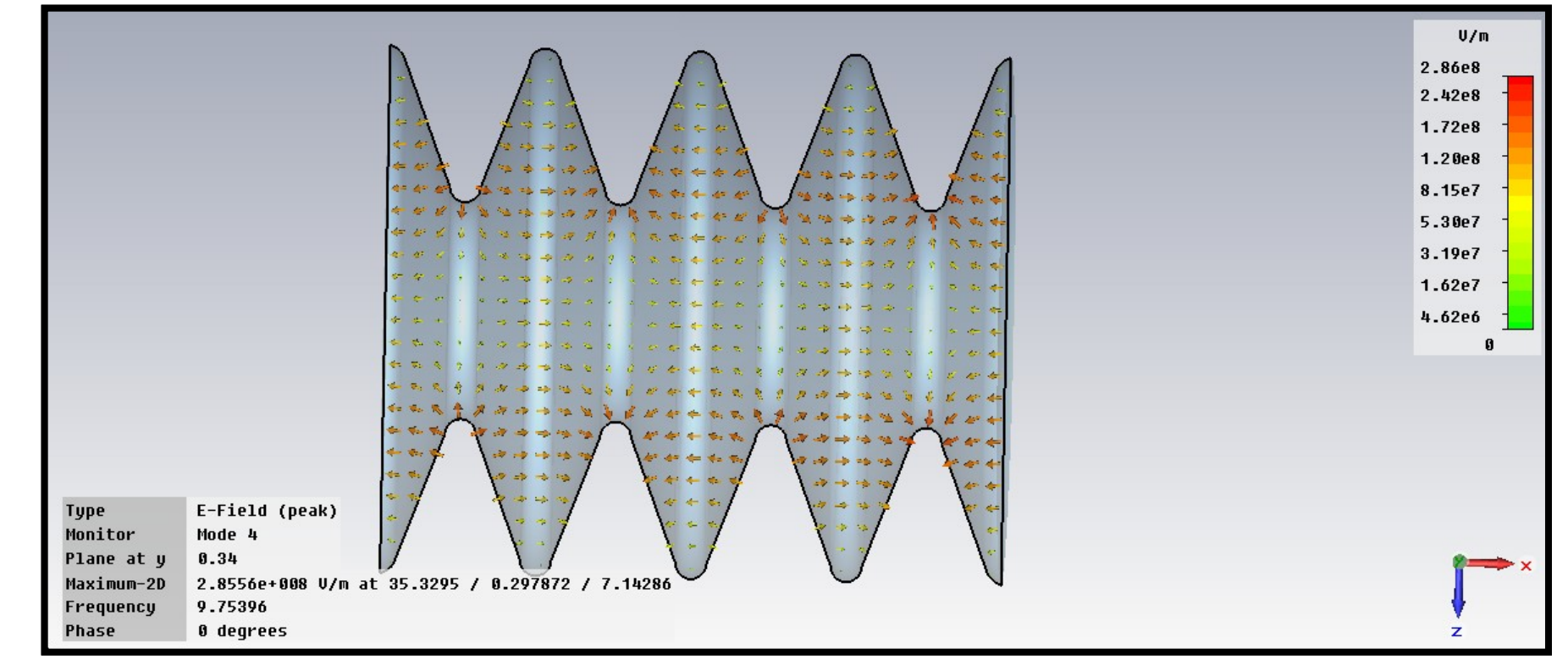


Figure 3

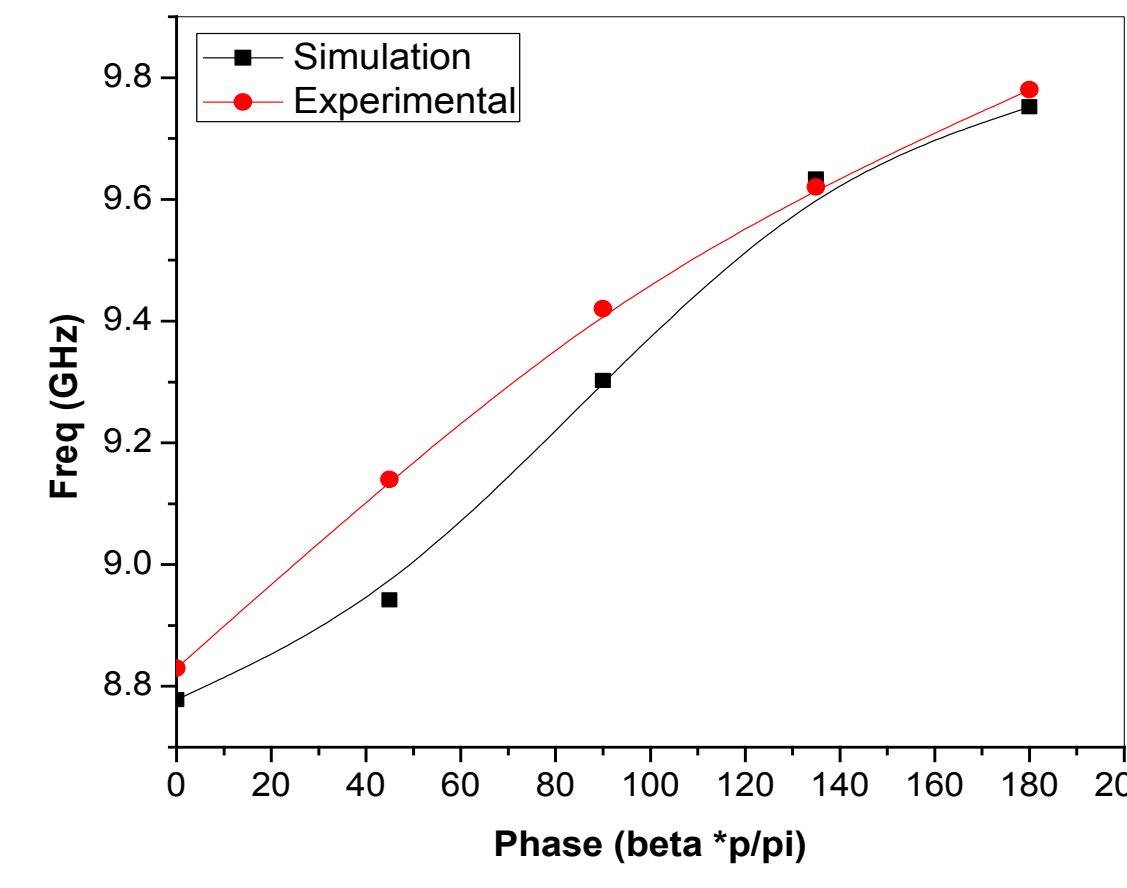


Figure 4

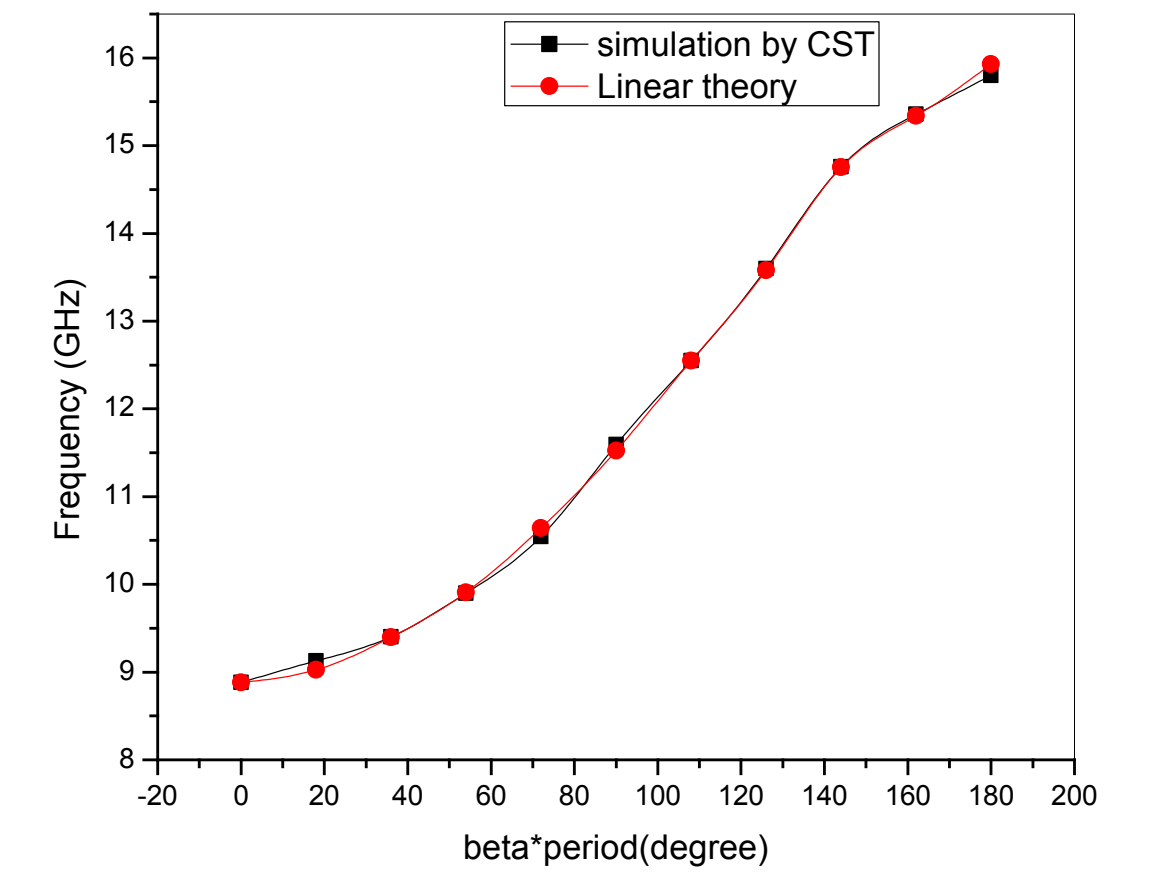


Figure 5

## Discussion

When the structure was fabricated due to limitation in machining it was difficult to produce exact sinusoidal corrugation. Care has been taken to reproduce the same structure while designing it in CST for simulation.

The pass band frequency range for the fabricated structure is found to be 8.8 GHz to 9.8 GHz which is under X-Band as shown in figure 4. The error found in theory and simulation is less than 7%.

The code based on linear theory was also applied to the structure but it failed to give same results. When the code was used on another structure with low corrugation depth ( $r_1=0.1$  cm) the results matched well with the simulation results. This is due to the fact that when truncating the matrix we are losing higher order terms which become important as the corrugation depth increases. If we try to increase the rank of matrix by taking more elements it becomes intractable for computer calculation. Here we have truncated the infinite matrix to a  $3 \times 3$  size for dispersion measurement.

## Conclusion

The rippled wall slow wave structure for the X-Band PASOTRON has been fabricated and successfully tested for operation in X-Band.

The dispersion for the structure is estimated both by simulation and experiment and the results are in close agreement.

A code based on linear theory is developed and dispersion calculated from it. The dispersion results from the code matches with simulation results for small corrugation.

For large corrugation there is mismatch between code results and simulation results and the reasons for the same has been discussed.

## References

- J.M. Butler, D.M. Goebel, R. W. Schumacher, J. Hyman, J. Santoru, R.M. Watkins, R.J. Harvey, F.A. Dolezal, R.L. Eisenhart and A.J. Schneider, "Pasotron High-energy Microwave Source", CH3141-9/92/0000-0511\$01.00 © 1992 IEEE
- R.A. Case et al, "High Power BWO driven by a Relativistic Electron Beam", IEEE Trans. Plasma Sci. PS-13 pp. 559-562 (1985)
- W. Main, Y. Carmel, K. Ogura, J. Weaver and S. Watanabe "Cold test Measurements of a BWO Slow-Wave Structure" 0-7803-1203-1/93\$03.00 © 1993 IEEE
- A. W. Horsley and A. Pearson, "Measurement of dispersion and Interaction Impedance characteristics of Slow-Wave Structures by Resonance Methods" IEEE Transactions On Electron Devices, Vol. Ed-13, No. 12, December 1966
- S. J. Rao, S. Ghosh, P. K. Jain, and B. N. Basu "Nonresonant Perturbation Measurements on Dispersion and Interaction Impedance Characteristics of Helical Slow-Wave Structures" IEEE Transactions On Microwave Theory And Techniques, Vol. 45, No. 9, September 1997

## Acknowledgements

The work has been carried out under CSIR 12<sup>th</sup> VY Network Project. The authors wish to acknowledge the team members of Plasma Devices Technology at CSIR-CEERI, Pilani and Prof. K.P. Maheshwari from DAVV Indore (formerly) for useful discussion.